

Two-Photon Decays of Mesons in a Relativistic Quark Model

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Abstract

We present a relativistic calculation of two-photon decays for heavy and light mesons in the framework of the Salpeter equation for quark-antiquark states. The meson-photon-photon vertex is computed by reconstructing the Bethe-Salpeter vertex function and evaluating the four-dimensional Feynman diagram with off-shell quark amplitudes.

The two-photon width for light and heavy quarkonia up to spin equal six are calculated with different parameter sets taken from the literature thus giving a complete overview on the mesonic two-photon physics. We find that relativistic effects including the negative energy components of the wave function are important for any two-photon width - even for heavy quarkonia - yielding a remarkable agreement with available data.

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I. INTRODUCTION

The decay into two photons is considered as an interesting experimental playground in the mesonic physics of the near future. New data will become available not only from existing experiments at CLEO, LEP, TRISTAN and VEPP, see [1] for a recent review on present experimental data, but from upgrades such as LEP2000 and new facilities as DAΦNE or the Cornell, KEK and SLAC B-factories.

The two-photon decay of mesons can be used to identify the flavor of quark-anti-quark states, but it may also discriminate between conventional mesons and glueballs or hybrids. Especially in the isoscalar spectrum of light scalar and tensor mesons a variety of resonances has been found which does not fit into the conventional $q\bar{q}$ nonets. A quantitative theoretical and experimental understanding of the two-photon decays may help to interpret the meson spectrum.

Most of the few theoretical results, however, contain large uncertainties in the overall scale of two-photon widths, especially in the light quark sector. The reason is, that the corrections to nonrelativistic decay formulae are large even in case of heavy quarkonia [2,3,4]. The situation is most dramatic in the case of the pion, where a nonrelativistic ansatz fails by orders of magnitude. Relativistic corrections up to now are based on the Feynman diagram for a *free* quark-antiquark pair annihilating into two photons. Bergström et al. [3] keep only positive energy components of the intermediate quark propagator and use nonrelativistic wave functions. Their results especially in the light meson sector do not agree well with experimental data. Li et al. [4] use a field-theoretical approach from the decay of p-wave positronium. From simple harmonic oscillator wave functions they calculate relativistic corrections to the nonrelativistic ratio 15/4 between the scalar and tensor decay width. The authors stress the necessity for a complete study within quark models, which give a good description of meson spectroscopy.

The most complete references on the calculation of two-photon widths are those of Godfrey and Isgur [5] and of Ackleh and Barnes [6]. They use wave functions, which follow from a semi- or nonrelativistic calculation of the meson mass spectra, which again is multiplied with the Feynman transition amplitude for a *free* quark-antiquark pair annihilating into two photons. As it stands, this would, however, not at all give the correct dependence of the widths on the meson bound-state mass. Therefore for instance in the case of pseudoscalar mesons an additional phenomenological factor $(M_{exp}/M_{ref})^3$ is introduced, where M_{exp} is the experimental (or calculated) bound-state mass and M_{ref} is a reference mass, which in [5,6] is taken to be the rest plus kinetic energy of the quark-antiquark pair in the meson (“Mock-meson mass”). This phenomenological input is necessary, because the transition is calculated between free quarks, which especially in the case of a deeply bound such as the pion is a very crude approximation. However, it seems that such a mass dependence is a crucial input, as most of their results agree well with existing data. As pointed out by the authors, there is considerable arbitrariness and lack of a stringent derivation in the choice of the reference mass, and hence a corresponding uncertainty in the overall

scale of the decay rates.

With this background we present a calculation of two-photon widths in the framework of the instantaneous Bethe-Salpeter equation, which has several improvements as compared to the existing theoretical estimates. On the one hand we use the Salpeter equation for the calculation of the meson mass spectrum, which includes the negative energy components of the wave function. This allows for a relativistic normalization of quark-antiquark amplitudes given by Salpeter [7], which has a reasonable limit for deeply bound states. On the other hand we calculate the decay matrix element in the framework of the Mandelstam formalism, so that the dependence of the quark-antiquark relative *four*-momentum is accounted for. In that way the transition amplitude from the quark-antiquark state into two photons can be formulated for *off-shell* quarks. Taken together this leads to the correct mass dependence of the decay widths and thus eliminates additional phenomenological input of previous calculations.

The formalism is applied to a complete calculation of light and heavy mesons up to spin equal six, for both singlet and triplet states. The comparison of various model parameters given in the literature will lead to results, which are relatively model independent or at least give a theoretical error, and therefore may be used as a basis for experimental investigations and their interpretation.

II. RELATIVISTIC CALCULATION OF TWO-PHOTON WIDTH IN THE FRAMEWORK OF THE SALPETER EQUATION

A. Spectra and Two-photon Widths in the Salpeter Formalism

Starting point in our calculation of the two-photon decays is the Bethe-Salpeter equation in its instantaneous approximation (Salpeter equation). Its advantage with respect to nonrelativistic models are the inclusion of the full Dirac-structure and negative energy components in the wave function, which should be important for light quarkonia. Moreover, the instantaneous interaction allows to incorporate a phenomenological confinement in the form of a linearly rising potential. The equation for the Bethe-Salpeter amplitude $\chi(p)$ in the rest frame of the meson reads

$$S^{-1}(M/2 + p) \chi(p) S^{-1}(-M/2 + p) = \int \frac{d^4 p'}{(2\pi)^4} [-i V(\vec{p}, \vec{p}') \chi(p')] \quad (1)$$

where we have used an instantaneous interaction $V(\vec{p}, \vec{p}')$ and the quark propagators $S(p_i)$. Defining the equal time amplitude $\Phi(\vec{p}) := \int dp_0 / 2\pi \chi(p_0, \vec{p})$, we arrive at the well known Salpeter equation

$$\begin{aligned} M \Phi(\vec{p}) = & H(\vec{p}) \Phi(\vec{p}) - \Phi(\vec{p}) H(-\vec{p}) \\ & - \Lambda^+(\vec{p}) \gamma^0 \int \frac{d^3 p'}{(2\pi)^3} [V(\vec{p}, \vec{p}') \Phi(\vec{p}')] \gamma^0 \Lambda^-(\vec{p}) \end{aligned} \quad (2)$$

$$+ \Lambda^-(\vec{p}) \gamma^0 \int \frac{d^3 p'}{(2\pi)^3} [V(\vec{p}, \vec{p}') \Phi(\vec{p}')] \gamma^0 \Lambda^+(-\vec{p})$$

with the projectors $\Lambda^\pm = (\omega \pm H)/(2\omega)$, the Dirac Hamiltonian $H(\vec{p}) = \gamma^0(\vec{\gamma}\vec{p} + m)$ and $\omega = (m^2 + \vec{p}^2)^{1/2}$ with m the quark mass. The Salpeter equation is an eigenvalue equation for the bound-state mass M and can be solved numerically, see for instance [8,9,10]. It is important to note, that the normalization of the Salpeter amplitudes [7]

$$\int \frac{d^3 p}{(2\pi)^3} \text{tr} \left\{ \Phi^\dagger(\vec{p}) \Lambda^+(\vec{p}) \Phi(\vec{p}) \Lambda^-(-\vec{p}) - \Phi^\dagger(\vec{p}) \Lambda^-(\vec{p}) \Phi(\vec{p}) \Lambda^+(-\vec{p}) \right\} = 2M \quad (3)$$

is mandatory to obtain a reasonable dependence on the bound-state mass M .

The Bethe-Salpeter amplitude $\chi(p)$ is needed to calculate current matrix elements of the corresponding bound-states in the Mandelstam formalism [11]. As has been pointed out in [8], equation (1) allows for the reconstruction of the Bethe-Salpeter amplitude $\chi(p)$ in the rest frame from the equal time amplitude $\Phi(\vec{p})$ as

$$\chi(p) = S(M/2 + p) \int \frac{d^3 p'}{(2\pi)^3} [-i V(\vec{p}, \vec{p}') \Phi(\vec{p}')] S(-M/2 + p) \quad (4)$$

The transition amplitude for a meson with mass M decaying into two photons with momenta k and \tilde{k} and polarization vectors $\varepsilon_1, \varepsilon_2$ then follows from the Mandelstam formalism as

$$T = -i\sqrt{3} (ie_q)^2 \int \frac{d^4 p}{(2\pi)^4} \text{tr} \left\{ \chi(p) \left(\not{\varepsilon}_2 S(\frac{M}{2} + p - k) \not{\varepsilon}_1 + \not{\varepsilon}_1 S(\frac{M}{2} + p - \tilde{k}) \not{\varepsilon}_2 \right) \right\} \quad (5)$$

so that we find for the decay width

$$\Gamma(M \rightarrow \gamma\gamma) = \frac{3\pi}{2} \frac{\alpha^2}{M} \frac{1}{2J+1} \sum_{M_J, \lambda_1, \lambda_2} \left| \int \frac{d^4 p}{(2\pi)^4} \frac{e_q^4}{e^4} \text{tr} \left\{ \chi_{M_J}(p) \left(\not{\varepsilon}_2 S(\frac{M}{2} + p - k) \not{\varepsilon}_1 + \not{\varepsilon}_1 S(\frac{M}{2} + p - \tilde{k}) \not{\varepsilon}_2 \right) \right\} \right|^2 \quad (6)$$

where λ_1, λ_2 are the polarizations of the two photons, J and M_J are the spin quantum numbers of the decaying meson and k is fixed for example to the positive z-direction.

As for the numerical treatment, we have expanded the radial basis functions in a set of twenty Laguerre functions in momentum space. The results for both masses and two-photon widths are found to be stable with respect to a variation of the number of basis states and to the scale of the basis functions.

B. Quark-Antiquark Interaction

The most successful ansatz in parameterizing quark confinement in hadron spectroscopy has been a linear potential, see for instance Godfrey and Isgur [5] for a

detailed study of mesons as quark-antiquark states. Motivated from lattice calculations for static quarks, its spin structure is usually taken to be scalar. This however, must not be true for light quarks. We therefore use a confinement spin structure

$$[V_C(\vec{p}, \vec{p}') \Phi(\vec{p}')] = \mathcal{V}_C(\vec{p} - \vec{p}') \left[(1 - x) \Phi(\vec{p}') - x \gamma^0 \Phi(\vec{p}') \gamma^0 \right] \quad (7)$$

where $x = 0$ is the scalar confinement used in non- and semi-relativistic quark models and $x = 1$ is a timelike vector spin structure used in the Salpeter model V of Resag and Münz [10] for heavy quarkonia. The scalar function \mathcal{V}_C is given in coordinate space by the commonly used linearly rising potential $\mathcal{V}_C(r) = a + br$.

For heavy quarkonia one usually takes beside the linear confinement potential a residual interaction coming from the one-gluon-exchange (OGE). Its implementation with a running coupling constant in the Salpeter formalism has been discussed extensively in [10]. In order to obtain results which are as far as possible model independent, we compare in the following several parameter sets for heavy quarkonia found in the literature. We investigate the parameter set of a nonrelativistic calculation for both mesons and baryon of Bhadhuri et al. [12] with a fixed coupling constant and the semi-relativistic ansatz of Godfrey and Isgur for the meson spectrum and decays [5] with running coupling constant. A fixed coupling constant α_s , however, leads in part to divergent Salpeter amplitudes for $r \rightarrow 0$ [13]. For a running coupling constant this phenomenon is less pronounced, but still present. We therefore use a regularization procedure by cutting off the OGE potential smoothly for $r < r_0$ [10]. In the model of [12] we use $r_0 = 0.1 \text{ fm}$, in [5] we use their parameter from the smearing function, which essentially is an equivalent regularization procedure. The meson masses calculated in the Salpeter model differ substantially only for the η_c in [12], which lies 100 MeV lower than in the nonrelativistic calculation. In addition to the parameters of this non- and semi-relativistic quark model we compare the Salpeter model [10] with timelike vector confinement structure for heavy quarkonia.

For light quarkonia with u, d and s quarks the situation is different with the following respects. We can no longer use parameter sets of non- or semi-relativistic quark models, as the dynamics including the negative energy components plays a quantitative effect. Moreover, the Salpeter equation with a purely scalar confinement leads to an instability [14,15], which becomes prominent in the case of light mesons. We therefore use a confinement structure with $x = 0.5$ to find reliable solutions for light quarkonia and to describe the spin-orbit splitting of the triplet p-wave mesons as good as possible.

In addition, the Salpeter equation does not allow to reproduce the masses of the light and heavy quarkonia with the same confinement strength, as can be done in a semi-relativistic ansatz [5]. The radial excitations of the Ψ and Υ states require a confinement strength of typically $b^{c,b} \approx 1300 \text{ MeV/fm}$ [10,16], whereas the description of the Regge trajectories for isovector and strange mesons requires $b^{u,d,s} \approx 1900 \text{ MeV/fm}$ [16,17]. Unfortunately the Salpeter models for light mesons with OGE interaction that have been presented so far use a fixed coupling constant α_s [16] with the above mentioned divergent Salpeter amplitudes for $r \rightarrow 0$, which are

not suitable for our purpose. We thus present two fits for light quarkonia, which both reproduce the experimental mass spectrum except for the η meson sector. We use a semi-relativistic value for the nonstrange quark mass of 220 MeV in the fit OGE-SRM (*Semi-Relativistic quark Mass*) and a nonstrange mass of 330 MeV common to nonrelativistic calculations in OGE-NRM (*Non-Relativistic quark Mass*). As shown in [6], the quark mass dependence of the two-photon widths is the most prominent one, so that the two models will estimate the theoretical error. As it is not possible to formulate the one-gluon-exchange interaction gauge invariantly, we will use the Feynman gauge in the first and the Coulomb gauge in the latter parameter set. Again we use the regularization parameter of $r_0 = 0.1$ fm.

In the light quark sector, however, 't Hooft [18] has derived another QCD inspired residual $q\bar{q}$ interaction coming from instanton effects. It has been proposed to solve the $U_A(1)$ problem and was successfully used in a nonrelativistic setup for the description of the π - η splitting and the η - η' mixing [19]. In the framework of the Salpeter equation it acts in addition on scalar mesons and gives an interesting interpretation of the scalar mixing and splitting [17]. We will therefore also present two-photon widths in this model for comparison. The parameter sets and interaction type of all the above mentioned models are given in Table I.

III. RESULTS AND DISCUSSION

A. Two-photon Decays of Heavy Quarkonia

In Tables II and III we present our numerical results for the two-photon widths of the charmonium and bottomonium singlet and triplet states up to spin equal four which are allowed by the Yang-theorem [20]. We used the three above mentioned parameter sets to extract a theoretical estimate including a “statistical” error, which of course includes only the parameter dependence. As the width is decreasing very rapidly with increasing total spin for these (almost) nonrelativistic systems, higher angular excitations are no more interesting. These calculations complete and improve results from other authors on spin singlet states [6] and on scalar and tensor mesons [3]. The conceptional improvement with respect to previous work comes from the use of relativistic amplitudes, from their Salpeter normalization and from the off-shell treatment of the quarks, which allows to renounce on phenomenological “Mock-meson” correction factors, being more and more uncertain for angular excited states. To estimate the effect of these improvements, we have compared in table IV our widths of the triplet charmonium and bottomonium states with those of the work of Bergström et al. [3] using the same parameter set of [12]. For the scalar mesons their full result, which is a factor of two smaller than the nonrelativistic ansatz, agrees with ours almost quantitatively. The tensor states, however, seem to be more sensitive to the above discussed relativistic effects, as our results differ from those of [3] by a factor of two.

Our calculation shows good agreement with available experimental data [21] on the η_c , χ_0 and χ_2 , especially with a recent measurement from the CLEO collaboration [22]. The results agree well with those of the theoretical prediction of Ackleh and Barnes [6] for the singlet bottomonium states. Their widths of singlet charmonium, however, are somewhat larger.

As concerned to future investigations for instance in $\gamma\gamma$ collision experiments our calculations suggest a strong coupling of the ground state and radial excitations of the η_c and χ_0 charmonium states. Especially the $\eta'_c(3590)$ with a predicted width of $\Gamma(\eta'_c \rightarrow \gamma\gamma) = 1400 \pm 400$ eV could probably be confirmed in a $\gamma\gamma$ experimental setup, as it lies below the $D\bar{D}$ threshold. In the bottomonium sector only the η_b has an appreciable two-photon width of 200 eV beside its radial excitations lying below the $B\bar{B}$ threshold with a width of around 100 eV.

B. Two-photon Decays of Light Quarkonia

In the light quark sector a relativistic treatment of meson decay observables becomes mandatory, see for instance [23] for an almost quantitative description of the electromagnetic decays and form factors of the light ground state mesons.

One of the major conceptional improvements as compared to previous models is the dependence of the two-photon width on the bound-state mass M . If one uses the simplest possible Lagrangian for the coupling of a pseudoscalar field ϕ to two photons

$$\mathcal{L} = \frac{1}{2} g \phi F_{\mu\nu} \tilde{F}^{\mu\nu} \quad (8)$$

one finds for the width [6]

$$\Gamma(M \rightarrow \gamma\gamma) = \frac{1}{64\pi} g^2 M^3. \quad (9)$$

To check this behavior numerically, we varied the offset a of the confinement interaction, so that we can calculate the two-photon width of the pion as a function of the resulting bound-state mass. In the mass region between 70 MeV and 700 MeV the two-photon width can be reproduced almost quantitatively by the function

$$\Gamma(M \rightarrow \gamma\gamma) = 2.95 \cdot 10^{-6} \text{eV} \left(\frac{M}{\text{MeV}} \right)^{2.95} \quad (10)$$

The M -dependence that is expected from a simple Lagrangian thus naturally emerges from the Salpeter formalism, in contrast to nonrelativistic calculations.

In Tables V and VI we compared our results for the fit with the instanton induced interaction by Klempt et al. [17] and the two fits with OGE interaction to the available experimental data. Flavor mixing is included only for the mesons with spin zero in [17]. In the other cases the nonstrange isoscalar states in Table VI can be identified from their isovector partners in Table V.

There exist only few data for mesons, which are well established $q\bar{q}$ resonances. Amongst them are the tensor mesons a_2 , f_2 and f_2' where our model predictions are in almost quantitative agreement with the experimental data, which we consider a main success of this work. In addition, our calculation shows that the nonstrange radial excitations of the a_2 and the f_2 , namely the f-wave at around 1800 MeV and the p-wave at around 1900 MeV, one of them probably corresponding to the $f_2(1810)$, both have a two-photon width of approximately 1 keV and therefore could be found also experimentally. This in turn would help to clarify the nature of the various tensor mesons and probably allow to identify glueball candidates in this sector.

As for the pseudoscalar ground states π and η , it has been shown in [9,23], that a quantitative description of these mesons requires a very light quark mass of 170 MeV, with an ansatz which accounts for the π - η splitting and the η - η' mixing as given by instanton effects. Nevertheless the quark masses that have been used here give a better mass spectrum with reasonable two-photon widths for the π , and also for the η and η' meson in the model of Klempt et al. [17]. The OGE interaction of course does not give the η - η' mixing so that these values are far off mainly due to the wrong bound-state mass.

Our results may also help to clarify the nature of the higher lying η resonances. The situation again is different for a conventional OGE or an instanton induced interaction. The former predicts an $n\bar{n}$ state at around 1350 MeV with a two-photon width of 650 ± 300 eV and an $s\bar{s}$ state at around 1700 MeV with a very small width. The instanton induced interaction on the other hand predicts an almost SU(3) flavor octet at 1550 MeV with a small width of 40 eV and a singlet at 1800 MeV with a large width of 500 eV. An experimental investigation of the η resonances, especially the $\eta(1295)$, could probably reveal the relevance of instanton effects for these mesons.

The interpretation of the scalar meson spectra is nontrivial due to the appearance of many states which do not fit into conventional $q\bar{q}$ nonets – see for instance [17] for a discussion in the framework of a relativistic quark model with instanton induced $q\bar{q}$ -forces. However, both the OGE and the instanton induced interaction predict a f_0 $q\bar{q}$ ground state with a two-photon width of around 1500 eV, which is a factor of three large than the experimental value of 560 ± 110 eV for the $f_0(980)$, which strengthens its interpretation as a $K\bar{K}$ molecule, see also [24]. Our large two-photon would fit much better to the broad $f_0(1300)$ resonance, being probably the $q\bar{q}$ ground state.

In the case of the $a_0(980)$ we find a discrepancy of a factor of two between our OGE calculations and the experimental value – assuming that its total width is almost equal to its partial width into $\pi\eta$. The experimental evidence for a $K\bar{K}$ molecule is thus less stringent for the a_0 than for the f_0 .

As for the comparison with nonrelativistic results, we find that the ratio of $15/4=3.75$ between the 0^{++} and the 2^{++} width is dramatically reduced for light mesons – after phase space correction – to around 1.3 ± 0.2 , a phenomenon that has already been predicted by [4].

One of the unsolved problems in the physics of mesonic two-photon decays is the large width of the $\pi_2(1670)$. The nonrelativistic ansatz of Anderson et al. [25] yields

very small widths of a few eV. All the parameter sets used in our Salpeter model predict a width around 100 eV, which is in agreement with the calculation of [6], whereas the particle data booklet gives 1350 ± 260 eV. However, this value comes from an incoherent analysis of different π_2 decay modes, whereas it has been convincingly argued in [26] to use a constructive interference between the $f_2\pi$ and $\rho\pi$ decay mode, which yields $800 \pm 300 \pm 120$ eV. Recent results from [27] also give a smaller value of 470^{+140}_{-180} eV, which is almost compatible with the quark model calculations. A possible explanation of the discrepancy between theory and experiment would be an interference between the π_2 with the second a_2 state, which up to now has not been found, but within quark models should have a mass of around 1800 MeV. As in our calculation it has a width of 300 eV, it could have influenced the two experiments cited by the Particle Data Group [21], as none of them has performed an analysis of angular distributions.

Finally we would like to emphasize those mesons which can most probably be seen in $\gamma\gamma$ experiments because of their large two-photon width, namely the a_2 and the nonstrange f_2 including their radial excitations, the a_0 and f_0 ground states and the radial excitation of the nonstrange η . However, it has been found that also angular excited mesons such as the nonstrange f_3 and f_4 or the η_2 are also promising candidates.

IV. SUMMARY AND CONCLUSION

We have presented a relativistic quark model calculation including a complete numerical study of the annihilation on quark-antiquark states into two photons. Several conceptual aspects have been improved with respect to previous theoretical work: The meson mass spectra are calculated in the framework of the Salpeter equation, which includes negative energy components of the meson wave function. These allow for a normalization condition given by Salpeter, which is adequate for states with large binding energy, as is the case for light quarkonia.

The meson to two-photon transition matrix element is calculated in the Mandelstam formalism. To this aim we use four-dimensional Bethe-Salpeter amplitudes which have been reconstructed from the Salpeter amplitudes that are the solutions of the bound-state mass eigenvalue equation. In contrast to previous work we thus do not put the quarks on mass-shell in the transition matrix element, but evaluate the four-dimensional Feynman diagram including the dependence on the relative energy. Together with the Salpeter normalization this leads for instance in the case of pseudoscalars to a reasonable behavior $\Gamma_{M \rightarrow \gamma\gamma} \sim M^3$ as expected from a simple Lagrangian. Therefore our model does not rely on the additional “Mock-meson” factor which has been necessary in previous work and contains an additional theoretical uncertainty in the scale of the two-photon widths.

In the numerical study that has been presented in the second part of this paper we have compared the parameter sets of non- and semi-relativistic quark model and

of Salpeter models for the meson spectra. The results for most of the two-photon widths are found to be stable against parameter changes. From the various models we have extracted theoretical predictions including error estimates for the widths of singlet and triplet quark-antiquark states up to spin six.

As our results agree almost quantitatively with many well established $q\bar{q}$ resonances such as the η_c , χ_{c0} , χ_{c2} or the light tensor nonet a_2 , f_2 and f'_2 , it is hoped that this complete numerical study can be used as a guideline for present and future experiments. In particular, we suggest that the most successful candidates for an experimental investigation in the heavy quark sector are the η_c and χ_0 including their radial excitations. In the light quark sector, the nonstrange f_2 and its radial excitations couple most strongly to two photons. Both the second p- and the first f-wave state have a width of around 1 keV and therefore may be measured experimentally. This in turn would clarify the f_2 meson mass spectrum and could probably allow the identification of tensor glueballs, one of the most interesting QCD phenomena.

REFERENCES

- [1] D. Morgan, M. R. Pennington, M. R. Whalley, J. Phys. **G20** Suppl. 8A (1994) A1.
- [2] L. Bergström, H. Snellman, G. Tengstrand, Phys. Lett. **B82** (1979) 419.
- [3] L. Bergström, G. Hulth, H. Snellman, Z. Phys. **C16** (1983) 263.
- [4] Z. P. Li, F. E. Close, T. Barnes, Phys. Rev. **D43** (1991) 2161.
- [5] S. Godfrey, N. Isgur, Phys. Rev. **D32** (1985) 189.
- [6] E. S. Ackleh, T. Barnes, Phys. Rev. **D45** (1992) 232.
- [7] E. E. Salpeter, Phys. Rev. **87** (1952) 328.
- [8] J. Resag, C. R. Münz, B. C. Metsch, H. R. Petry, Nucl. Phys. **A578** (1994) 397.
- [9] C. R. Münz, J. Resag, B. C. Metsch, H. R. Petry, Nucl. Phys. **A578** (1994) 418.
- [10] J. Resag, C. R. Münz, Nucl. Phys. **A590** (1995) 735.
- [11] S. Mandelstam, Proc. Roy. Soc. **233** (1955) 248.
- [12] R. K. Bhaduri, L. E. Cohler, Y. Nogami, Nuovo Cimento **65A** (1981) 376.
- [13] T. Murota, Progr. Theor. Phys. **69** (1983) 181 , *ibid.* 1498.
- [14] A. Archvadze, M. Chachkhunashvili, T. Kopaleishvili, Few-Body Systems **14** (1993) 53.
- [15] J. Parramore, J. Piekarewicz, Nucl. Phys. **A585** (1995) 705.
- [16] J. Parramore, H. C. Jean, J. Piekarewicz, Florida State University FSU-SCRI-95-105, nucl-th/9510024 (1995).
- [17] E. Klempt, B. C. Metsch, C. R. Münz, H. R. Petry, Phys. Lett. **B361** (1995) 160.
- [18] G. 't Hooft, Phys. Rev. **D14** (1976) 3432.
- [19] W. H. Blask, U. Bohn, M. G. Huber, B. C. Metsch, H. R. Petry, Z. Phys. **A337** (1990) 327.
- [20] C. N. Yang, Phys. Rev. **77** (1950) 242.
- [21] Particle Data Group, Phys. Rev. **D50** (1994) 1173.
- [22] V. Savinov, R. Fulton (CLEO collaboration), Contribution to the PHOTON95 conference, Sheffield (1995), hep-ex/9507006.
- [23] C. R. Münz, J. Resag, B. C. Metsch, H. R. Petry, Phys. Rev. **C52** (1995) 2110.
- [24] T. Barnes, Phys. Lett. **B165** (1985) 434.
- [25] J. D. Anderson, M. H. Austern, R. N. Cahn, Phys. Rev. **D43** (1991) 2094.
- [26] H. J. Behrend et al. (CELLO Collaboration) Z. Phys. **C46** (1990) 583.
- [27] E. Kriznic, private communication in [1].

TABLES

TABLE I. Parameters and interaction type of the different models

Model	Quark masses				Confinement Spin structure	Residual		α_{sat}
	m_n	m_s	m_c	m_b		a	b	
Godfrey & Isgur [5]	—	—	1628	4977	Scalar	-253	914	OGE-Coul-Run 0.60
Bhaduri et al. [12]	—	—	1870	5259	Scalar	-913	941	OGE-Coul-Fix 0.39
Resag & Münz V [10]	—	—	1631	5005	Vector	-640	1291	OGE-Coul-Run 0.365
Klempt et al. [17]	306	503	—	—	Scal.+Vect.	-1751	2076	't Hooft -
OGE-SRM	220	468	—	—	Scal.+Vect.	-1273	1856	OGE-Feyn-Run 0.224
OGE-NRM	330	567	—	—	Scal.+Vect.	-1418	1956	OGE-Coul-Run 0.303

TABLE II. Calculated $\gamma\gamma$ -widths of charmonium states in eV for several models and the theoretical estimate compared to experimental data (meson masses in GeV are given in parenthesis).

	J^{PC}	Godfrey [5]	Bhaduri [12]	Resag V [10]	Theor. Estimate	Exp.
$\eta_c(2.98)$	0^{-+}	3690 (2.97)	2960 (2.88)	3820 (2.98)	3500 ± 400	7000^{+2000}_{-1700} [21] 4300 ± 1900 [22]
$\eta'_c(3.59)$	0^{-+}	1400 (3.62)	1000 (3.57)	1730 (3.67)	1380 ± 300	
η''_c	0^{-+}	930 (4.02)	657 (3.99)	1220 (4.16)	940 ± 230	
η'''_c	0^{-+}	720 (4.320)	502 (4.32)	979 (4.58)	730 ± 200	
η''''_c	0^{-+}	610 (4.580)	421 (4.60)	831 (4.94)	620 ± 170	
η_{c2}	2^{-+}	9.1 (3.77)	4.1 (3.79)	13.6 (3.84)	9 ± 4	
η'_{c2}	2^{-+}	12.1 (4.11)	7.2 (4.14)	20.2 (4.28)	13 ± 6	
η_{c4}	4^{-+}	0.108 (4.16)	0.028 (4.19)	0.304 (4.40)	0.15 ± 0.12	
$\chi_{c0}(3.41)$	0^{++}	1290 (3.45)	1270 (3.49)	1620 (3.40)	1390 ± 160	4000 ± 2800 [21] 1700 ± 800 [22]
χ'_{c0}	0^{++}	950 (3.88)	1140 (3.90)	1250 (3.94)	1110 ± 130	
χ''_{c0}	0^{++}	741 (4.20)	969 (4.23)	1020 (4.38)	910 ± 130	
$\chi_{c2}(3.55)$	2^{++}	459 (3.53)	259 (3.54)	601 (3.56)	440 ± 140	321 ± 95 [21] 700 ± 300 [22]
χ'_{c2}	2^{++}	449 (3.93)	317 (3.95)	684 (4.05)	480 ± 160	
χ''_{c2}	2^{++}	16.3 (3.99)	3.7 (4.02)	23.4 (4.09)	14 ± 8	
χ_{c3}	3^{++}	1.53 (4.00)	0.44 (4.02)	3.07 (4.13)	1.7 ± 1.1	
χ_{c4}	4^{++}	1.09 (3.99)	0.31 (4.01)	2.12 (4.19)	1.2 ± 0.8	

TABLE III. Calculated $\gamma\gamma$ -widths of bottomonium states in eV for several models.

	J^{PC}	Godfrey [5]	Bhaduri [12]	Resag V [10]	Theor. Estimate
η_b	0^{-+}	214 (9.47)	266 (9.37)	192 (9.44)	220 ± 40
η'_b	0^{-+}	121 (10.01)	95.0 (10.01)	116 (10.01)	110 ± 20
η''_b	0^{-+}	90.6 (10.35)	67.9 (10.36)	93.5 (10.39)	84 ± 12
η'''_b	0^{-+}	75.5 (10.62)	56.3 (10.63)	81.8 (10.72)	71 ± 11
η_{b2}	2^{-+}	41.6 meV (10.13)	28.3 meV (10.19)	51.3 meV (10.13)	40 ± 10
η'_{b2}	2^{-+}	69.8 meV (10.43)	52.3 (10.47)	96.2 meV (10.47)	73 ± 18
η_{b4}	4^{-+}	41.0 μ eV (10.51)	15.9 μ eV (10.54)	71.9 μ eV (10.56)	43 ± 23
χ_{b0} (9.86)	0^{++}	20.8 (9.89)	27.3 (9.91)	24.1 (9.83)	24 ± 3
χ'_{b0} (10.23)	0^{++}	22.7 (10.25)	26.9 (10.27)	27.3 (10.25)	26 ± 2
χ_{b2} (9.91)	2^{++}	5.14 (9.92)	5.26 (9.95)	6.45 (9.87)	5.6 ± 0.6
χ'_{b2} (10.27)	2^{++}	6.21 (10.27)	6.11 (10.30)	8.1 (10.28)	6.8 ± 1.0
χ_{b3}	3^{++}	1.57 meV (10.35)	0.72 meV (10.39)	2.41 meV (10.36)	1.6 ± 0.7
χ_{b4}	4^{++}	1.26 meV (10.35)	0.58 meV (10.39)	1.94 meV (10.37)	1.3 ± 0.6

TABLE IV. Comparison of the scalar and tensor charmonium and bottomonium widths with the results of Bergstöm et al. [3]

	J^{PC}	Salpeter-model	Bergstöm [3]	Bergstöm non-rel. [3]	Exp.
χ_{c0}	0^{++}	1270	1360	2960	1700 ± 800 [22]
χ_{c2}	2^{++}	259	498	789	321 ± 95 [21]
χ_{b0}	0^{++}	27.3	27.6	44.3	
χ_{b2}	2^{++}	5.26	9.35	11.8	

TABLE V. Calculated $\gamma\gamma$ -widths of isovector states in eV for a model with instanton induced [17] and with OGE interaction compared to experimental data (meson masses in MeV are given in parentheses).

	J^{PC}	Klempt [17]	OGE-NRM	OGE-SRM	Exp.
π (138)	0^{-+}	4.23 (140)	3.81 (140)	5.07 (140)	7.84 ± 0.56 [21]
π' (1300)	0^{-+}	151 (1360)	127 (1380)	355 (1330)	
π''	0^{-+}	2.00 (2010)	4.09 (2020)	7.47 (1920)	
π_2 (1670)	2^{-+}	94.2 (1630)	73.2 (1650)	129 (1560)	
					1350 ± 260 [21]
					470^{+140}_{-180} [27]
π'_2 (2100)	2^{-+}	12.0 (2160)	4.93 (2150)	2.48 (2020)	
π_4	4^{-+}	28.3 (2220)	23.6 (2260)	51.2 (2120)	
π_6	6^{-+}	10.4 (2680)	8.4 (2720)	20.8 (2550)	
a_0 (980)	0^{++}	1390 (1320)	640 (1080)	486 (1010)	
					$\gtrsim 240^{+80}_{-70}$ [21]
a'_0	0^{++}	386 (1930)	54.0 (1780)	28.5 (1680)	1040 ± 90 [21]
a''_0	0^{++}	291 (2420)	30.3 (2290)	11.8 (2140)	
a_2 (1320)	2^{++}	734 (1310)	766 (1330)	900 (1280)	
a'_2	2^{++}	374 (1880)	326 (1870)	376 (1740)	
a''_2	2^{++}	293 (1930)	320 (1930)	353 (1820)	
a_3	3^{++}	60.9 (1950)	56.2 (1970)	93.5 (1840)	
a_4 (2040)	4^{++}	50.4 (2010)	43.8 (2030)	87.9 (1910)	
a_5	5^{++}	15.8 (2460)	13.5 (2490)	27.9 (2320)	
a_6 (2450)	6^{++}	14.0 (2520)	11.2 (2530)	28.5 (2380)	

TABLE VI. Calculated $\gamma\gamma$ -widths of isoscalar light quarkonia in eV.

	J^{PC}	Klempt [17]	OGE-NRM	OGE-SRM	Exp. [21]
η (547)	0^{-+}	208 (530)	10.6 (142)	14.1 (138)	460 ± 40
η' (958)	0^{-+}	2330 (980)	39.5 (665)	45.8 (641)	4260 ± 190
η'' (1295)	0^{-+}	31.8 (1530)	350 (1390)	986 (1330)	
η'''	0^{-+}	499 (1810)	0.75 (1700)	0.27 (1660)	
η''''	0^{-+}	510 (2180)	11.4 (2020)	20.8 (1920)	
η'''''	0^{-+}	544 (2380)	5.7 (2350)	3.9 (2280)	
η_2	2^{-+}	262 (1630)	203 (1650)	358 (1560)	
η'_2	2^{-+}	9.89 (1860)	7.2 (1940)	10.7 (1870)	
η''_2	2^{-+}	33.4 (2160)	13.7 (2150)	6.9 (2020)	
η'''_2	2^{-+}	4.22 (2420)	2.7 (2490)	2.9 (2400)	
η_4	4^{-+}	78.7 (2220)	65.7 (2260)	142 (2120)	
η'_4	4^{-+}	1.77 (2480)	1.25 (2580)	22.0 (2470)	
η_6	6^{-+}	28.9 (2680)	23.4 (2720)	57.7 (2550)	
η'_6	6^{-+}	0.46 (2960)	10.3 (3050)	17.2 (2840)	
f_0 (980)	0^{++}	1750 (980)	1780 (1080)	1350 (1010)	560 ± 110
f_0 (1300)	0^{++}				5400 ± 2300
f'_0 (1590)	0^{++}	161 (1470)	192 (1400)	199 (1360)	
f''_0	0^{++}	15.3 (1780)	150 (1780)	79 (1680)	
f'''_0	0^{++}	29.2 (2110)	50.6 (2110)	45.5 (2050)	
f''''_0	0^{++}	42 (2310)	84.3 (2290)	32.7 (2140)	
f_2 (1270)	2^{++}	2040 (1310)	2130 (1330)	2500 (1280)	2440^{+320}_{-290}
f'_2 (1525)	2^{++}	121 (1525)	127 (1590)	146 (1540)	105 ± 17
f''_2 (1810)	2^{++}	1038 (1880)	906 (1870)	1040 (1740)	
f'''_2	2^{++}	815 (1930)	888 (1930)	982 (1820)	
f''''_2 (2010)	2^{++}	75.2 (2150)	83.8 (2220)	52.9 (2130)	
f'''''_2 (2300)	2^{++}	42.7 (2160)	33.7 (2230)	75.9 (2160)	
f_3	3^{++}	169 (1950)	156 (1970)	260 (1840)	
f'_3	3^{++}	5.38 (2190)	4.56 (2290)	6.60 (2190)	
f_4 (2050)	4^{++}	140 (2010)	122 (2030)	244 (1910)	
f'_4 (2220)	4^{++}	3.81 (2230)	3.07 (2330)	121 (2240)	
f_5	5^{++}	44.0 (2460)	37.4 (2490)	77.5 (2320)	
f'_5	5^{++}	0.94 (2730)	0.67 (2830)	1.15 (2700)	
f_6 (2510)	6^{++}	38.9 (2520)	31.0 (2530)	79.1 (2380)	
f'_6	6^{++}	0.654 (2770)	0.435 (2860)	0.827 (2740)	